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DYNAMIC DISINTEGRATION AND EXPANSION OF A LIQUID VOLUME

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The process of dispersing (disintegrating) a liquid volume and forming a droplet–air cloud during an explosion can be divided into the following [three] stages. The first – the propagation of an explosive wave which arises when the charge detonates – is essentially determined by an explosion in an unbounded liquid, and has been studied theoretically and experimentally [1-5].

The second stage starts when the shock wave propagating through the liquid reaches a free surface, reflects, and produces a rarefaction. The tensile stress which arises behind the rarefaction front leads to the intense production of vapor–gas bubbles – the cavitation phenomenon [3, 6-8]. The solution to the problem of bubble cavitation has been examined in [9-14]. In particular, the explosive loading of a cylindrical liquid layer has been studied in [15-18]. Depending on the magnitude of the specific energy release it has been established that either cavitation disintegration or hydromechanical perturbations (of the Rayleigh–Taylor type) can occur on the inner and outer surfaces of the liquid volume. A break-away disintegration, which has been observed experimentally [3, 19] is also possible. The second stage ends when the volume concentration of bubbles in the liquid reaches a critical value, at which an inversion process takes place: the bubble-filled liquid transforms to a droplet stage.

The third stage starts with the formation of a droplet–vapor mixture; as it moves it is blown by the reverse flow of the surrounding air on the outer boundary and by the detonation products on the inner boundary. The expansion of the initial finely dispersed particles or droplets and estimates of the dimension of the resultant droplet–air cloud is discussed in [20] and [21].

Here we study the problem of the explosive dispersion of a liquid volume, the subsequent expansion of the resultant droplet–air cloud [22] in spherical and cylindrical geometry, and propose an approximate numerical model. The problem is examined for large-scale phenomena when the relaxation time of the tensile stresses in the rarefaction wave are small compared to a characteristic hydrodynamic time scale.

1. We study a solid chemical explosive charge with an initial density ρ_{ex} and radius R_{0c} which is surrounded by a liquid layer with a radius R_{0k}. The initial liquid pressure is p_{01} with density ρ_{01}. The liquid is surrounded by infinite air with an initial pressure p_{02} and density ρ_{02}. In the spherical case it is assumed that at time $t = 0$ an energy W is instantaneously released in a volume $v_i = 4/3\pi R_{0c}^3$; the intial pressure p_i of the detonation products is found from [5]

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$$p_i = \frac{1}{8}\rho_{ex} D^2 \tag{1.1}$$

where D is the detonation wave velocity. The discussion is similar for the cylindrical case; however the initial energy determined is W/H, where H is the legnth of the explosive material and the liquid layer; that is, the calculation is done for a liquid ring.

The detonation of a solid explosive in a liquid layer surrounded by infinite air is described by a system of one-dimensional transient hydrodynamic equations, which in Lagrangian variables are written as follows [23, 24]:

$$\frac{\partial r}{\partial t} = u; \tag{1.2}$$

$$\frac{\partial u}{\partial t} = -V_0\left(\frac{r}{r_0}\right)^{n-1}\frac{\partial p}{\partial r_0}; \tag{1.3}$$

$$V = V_0\left(\frac{r}{r_0}\right)^{n-1}\frac{\partial r}{\partial r_0}; \tag{1.4}$$

$$\frac{\partial e}{\partial t} = -p\frac{\partial V}{\partial t}. \tag{1.5}$$

Here n is 2 or 3 for cylindrical or spherical geometry; r is the Eulerian coordinate; r_0 is the Lagrangian coordinate; $V = 1/\rho$ is the specific volume; p is the pressure; ρ is the density; u is the mass velocity; e is the internal energy per unit mass; and t is the time. We emphasize that Eqs. (1.2)-(1.4), along with the equation of state (1.6) given below, determine the dynamic characteristics of the explosion, and (1.5) determines the energetic characteristics of the explosive process.

After the explosion a (spherical or cylindrical) shock wave propagates through the liquid volume and will attenuate over time due to dissipation at the front and to geometric expansion. According to estimates [5, 25], a shock wave propagating through a liquid is a weak shock almost from the beginning; that is, its properties do not differ too much from those of an acoustic wave. At the wave front, the change in entropy is small and the compression of the liquid is insignificant. These discussions [5, 25] verify the use of the Tate equation [5] for the liquid equation of state:

$$p = \frac{\rho_{01}c_{liq}^2}{\gamma_{liq}}\left[\left(\frac{\rho_1}{\rho_{01}}\right)^{\gamma_{liq}} - 1\right], \tag{1.6}$$

where c_{liq} and γ_{liq} are the sound speed and adiabatic index of the liquid. We will assume that cavitation starts at p = 0. We introduce a unit [Heavyside] function which considers the effect of cavitation:

$$p = \frac{\rho_{01}c_{liq}^2}{\gamma_{liq}}\left[\left(\frac{\rho_1}{\rho_{01}}\right)^{\gamma_{liq}} - 1\right]\theta(\rho_1 - \rho_{01}).$$

We note that Eq. (1.6) corresponds to the limiting case where the tensile stress in the liquid relaxes instantaneously at zero pressure. This approximation is valid when the characteristic stress relaxation time in the liquid t_p (according to [9], $t_p = 10^{-7}$ sec) is much less than the characteristic hydrodynamic time $t_h \sim L/c_{liq}$, where L is the characteristic dimension of the liquid layer.

The air surrounding the liquid and the detonation products are considered to be an ideal gas, with the equation of state:

$$p = p_i(R_{0c}/R_c)^{3\gamma_d}.$$

Here the adiabatic index of the detonation products is $\gamma_d = \gamma_1 = 3$ for $p > p_k$ and $\gamma_d = \gamma_2 = 1.4$ for $p < p_k$, where p_k is a joining pressure for the double-branched adiabat. This pressure is determined in analogy with [2] by the equation

$$p_k = p_i \left\{ \frac{\gamma_2 - 1}{\gamma_1 - \gamma_2} \left[\frac{(\gamma_1 - 1) W}{p_i v_i} - 1 \right] \right\}^{3/2}.$$

The equation of state for air has the form $p = p_{02}(\rho_2/\rho_{02})^{\gamma_g}$, where γ_g is the adiabatic index of air.

The above equations of state and the equation of motion of the bubble-filled liquid correspond to an equilibrium two-phase method [26]: $T_1 = T_2 = T(p)$, $p_1 = p_2$, and $u_1 = u_2$, where the subscripts 1 and 2 indicate the liquid and the bubbles, respectively, and T is the temperature. The motion of the equilibrium mixture in this case is described by a system of equations for a single-phase continuous medium (liquid) with equations of state determined by the phase properties. If we assume that this liquid has a high boiling temperature and the pressure of the saturated vapor is much less than the background pressure, then the equilibrium system is expressed by a system with $p = 0$ in the cavitation zone.

2. The cavitation mechanism for breaking up the bubble-filled liquid into a droplet structure (inversion) has a topologically static character. It is assumed that due to the chaotic spatial distribution of the bubbles, there are regions were they bunch together and where their local volumetric concentration is high enough for them to agglomerate. This agglomeration of bubbles (clusterization) occurs as breaks along some surfaces which, by intersecting with each other, lead to the formation of a droplet structure.

According to [9], in the original liquid, the initial concentration of cavitation nuclei is 10^9-10^{12} m^{-3}, the volumetric content of the dispersed phase is $\alpha_2 \sim 10^{-11}$-10^{-4}, and the average radius of the nuclei is 10^{-8}-10^{-7} m. We will assume that the initial dimensional distribution of nuclei is exponential

$$dn = (N_0/r_1) \exp(-r/r_1)\, dr,$$

where N_0 is the concentration of growing bubble nuclei per unit volume, and $r_1 = 10^{-8}$ m is the initial dimension of a bubble nucleus in the liquid. Integration of this expression gives the number of bubbles involved in active growth

$$n = N_0 \exp(-r_*/r_1) = N_0 \exp(-\sigma/(|p_0| r_1)), \tag{2.1}$$

where we consider the fact that for a given tensile stress $|p_0|$, bubbles with dimensions larger larger than a threshold $r_0 \sim \sigma/|p_0|$ will grow irreversibly, where σ is the surface tension coefficient of the liquid. This expression for the cavitation threshold follows from an analysis of the equilibrium of a single bubble in a surrounding liquid with the aid of the Rayleigh–Lamb equation.

By examining this relaxation process of the tensile stresses due to the growth of the dispersed phase, it is not difficult to obtain an estimate for the amplitude of the tensile stresses as the product of the pressure gradient in the rarefaction wave, the sound speed in the liquid, and the characteristic relaxation time for the tensile stresses:

$$|p_0| = \frac{p_m}{L_p} c_{liq} t_p \simeq 1{,}6 n^{-1/3} \left(\frac{|p_0|}{\rho_{01} c_{liq}^2} \right)^{-1/6} \frac{p_m}{L_p}. \tag{2.2}$$

Here L_p is the width of the rarefaction front; p_m is the amplitude of the decaying shock wave; and n is the number of incipient bubbles per unit volume. By combining Eqs. (2.1) and (2.2) it is possible to make a quantitative estimate of the bubble density n as a function of the distance from the center of the explosion.

The average diameter d of the droplets, which form during the cavitational disintegration of the bubble-filled liquid, is determined by the density of growing bubbles n. From a topological statistical argument it follows that $d = An^{-1/3}$. An investigation of the droplet formation stage, based on the agglomeration of growing bubbles [27], shows that the proportionality coefficient A is on the order of unity. Within the framework of this approach, the dispersion composition of the droplet mixture can be controlled by changing the number of growing bubbles by changing the distribution of initial cavitation nuclei in the liquid (2.1).

By introducing the notation $X = (N_0/n)^{1/3} = d/d_0$ for $d_0 \simeq N_0^{-1/3}$, for ordinary liquids we obtain the transcendental equation

$$X^{6/7} \ln X = \alpha^{6/7},$$

where α depends on the parameters of the decaying wave and the properties of the liquid

$$\alpha = \left(\frac{\sigma}{3r_1\rho_{01}c_{liq}^2}\right)^{7/6} \frac{L_p\rho_{01}c_{liq}^2}{p_m d_0}.$$

3. Analysis of 1) the generation of the Rayleigh–Taylor instability at the boundary between the detonation products and the liquid and 2) the subsequent disintegration of the liquid, reduces to solving the problem of motion of a symmetric liquid layer with a consideration of liquid mass removal from its surface due to the growth and spallation of perturbed liquid microlayers.

We will examine the acceleration of a liquid layer by the gaseous detonation products. According to [26], the amplitude δ of harmonic perturbations of the interphase boundary increase with time as $\delta = \delta_0 \cdot \exp(It)$, where

$$I = \left[\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} a\frac{2\pi}{b} - \frac{\sigma}{\rho_1 + \rho_2}\frac{8\pi^3}{b^3}\right]^{1/2} \tag{3.1}$$

(a is the acceleration of the layer and b is the wavelength of the perturbation). In Eq. (3.1) a term inside the square brackets which corresponds to the destabilizing effect of the mutual tangential motion of the liquid and the gas is omitted. In determining the sign of the first term in (3.1), we must consider that $a > 0$ if the acceleration is directed from the gas to the liquid. That is, in our case, the acceleration of the layer is a destabilizing factor on its inner surface; here the corresponding term in Eq. (3.1) for the increment [I] is positive. The Rayleigh–Taylor instability of the interphase boundary occurs when the condition $I > 0$ is fulfilled for $b < 2r$, where r is a characteristic radius of the layer, which for $\rho_1 >> \rho_2$ gives a criterion for disintegration of the layer: $Bo = 4r^2\rho_1 a/\sigma > Bo_* \sim 4\pi^2$, where Bo is the Bond number. We assume that the spallation of the perturbing microlayer occurs at a time t_* when the amplitude of the perturbation is equal to half a wavelength. This does not contradict the assumption [26] that the dimension of the spalling droplet (or the amplitude of the perturbation) is on the order of a wavelength. The last condition makes it possible to determine the wavelength b_* which corresponds to the fastest growth of the perturbations:

$$b_* \approx r\sqrt{12\,Bo_*/Bo}.$$

Here the minimum time for disintegration – or the spallation of a single liquid microlayer – is written as

$$t_* = t(b_*) \approx \ln\left(\frac{b_*}{2\delta_0}\right)\left(\frac{\rho_1 r^3}{\sigma}\right)^{1/2}\left(\frac{Bo_*}{Bo}\right)^{3/4}. \tag{3.2}$$

If we assume that the logarithm in (3.2) is equal to unity and consider the fact that the acceleration of the liquid layer by the gas pressure p has the form $a = p/(\rho_1\Delta)$, where $\Delta(t)$ is the layer thickness, we obtain that the spallation time for the perturbed microlayer is not an explicit function of the radius r:

$$t_* = \pi^{3/2}\sigma\left(\frac{\rho_1^2\Delta^3}{p^3}\right)^{1/4}. \tag{3.3}$$

The motion of the liquid layer considering mass loss from its surface is described by the equation

$$n(p(t) - p_2)R_c^{n-1} = \rho_1(R_k^n - R_c^n)\ddot{R}_c + \dot{R}_c\dot{m}, \tag{3.4}$$

where $p(t) = p_i \cdot (R_{0c}/R_c)^{n\gamma}d$; m is the mass per unit length of the liquid layer; and the superscript dots indicate differentiation with respect to time. The mass loss rate $\dot{m}$ is related to the flux w of the liquid from the surface by the relationship $\dot{m} = -2\pi \cdot R_c w$ [$w = (1/2)\rho_1 \cdot \langle u\rangle$]. The velocity of the perturbed liquid surface changes from zero at the moment the perturbation starts to $u_* \approx I(b_*) \cdot b_*$ at the moment the perturbation spalls off. The average surface velocity is $\langle u\rangle = I(b_*) \cdot b_* \cdot (e + 1)/(2e)$; here it is assumed that the amplitude of the perturbation at the moment of spallation increases by a factor of e.

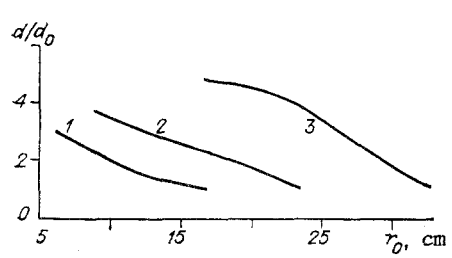

Fig. 1

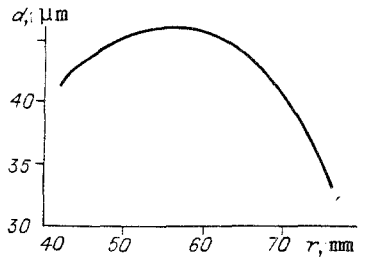

Fig. 2

4. In estimating the maximum dimension of the cloud, we do not consider the effect of the detonation products on the dispersion of the droplet mixture. We also do not consider the additional fragmentation of the flying droplets by the counterflow of the surrounding air. We assume that, when the inversion front ceases in the Lagrangian coordinate system, the pressure decreases inside and around the cloud, and an intense saturation begins in the surrounding air at a rate which gradually becomes constant. The region encompassed by saturation slowly (logarithmically) expands with time. We will neglect the logarithmic change of the saturation depth with time. In this approximation the boundary is taken to be impermeable to gas, the outwardly expanding cloud is only "mashed" on the side of the boundary, and the cloud itself acts on the surrounding air like a spherical piston. By assuming no permeation through the cloud boundary, we obtain a formula for the pressure drop

$$\Delta p = \rho_1 u_1^2, \tag{4.1}$$

where u_1 is the velocity of the permeation from relative to the initially immobile droplets. This pressure drop within the cloud represents the pressure at the cloud boundary, because within the cloud the pressure is zero ahead of the permeation front. On the other hand, the pressure at the cloud boundary is composed of the external (atmospheric) pressure and the dynamic pressure drop of the decelerating air. For a spherical piston and an incompressible liquid, the expression for the dynamic pressure drop can be written as

$$\Delta p = p_2 + \rho_2 \dot{R}^2, \tag{4.2}$$

which does not include the acceleration of the boundary and which agrees well with the results of [28] for a spherical piston in a compressible gas (R is the radius of the cloud boundary). By equating the boundary pressures (4.1) and (4.2) within and around the cloud, and also by considering the symmetry of the problem $\rho_1 \sim R^{-n}$, we obtain a differential equation for the cloud boundary in a dimensionless form

$$(1 - \dot{y})^2 Q_0 y^{-n} = \varepsilon + \dot{y}^2,$$

where $Q_0 = \rho_1(t = 0)/\rho_2$; $y = R/R_{0k}$; $\tau = t/t_0 = u_0 t/R_{0k}$; $M = u_0/c_g$; $\varepsilon = p_2/(\rho_2 u_0^2) = 1/(\gamma_g M^2)$; u_0 is the mass velocity of the air layer next to the free surface of the liquid at the moment the inversion front ceases; and c_g is the sound speed in unperturbed air. If the velocity of the cloud boundary is expressed explicitly, then it is easy to see that it goes to zero when $\varepsilon \cdot y^n \cdot Q_0^{-1} = 1$ and this immediately gives the maximum expansion radius R_m of the cloud:

$$\frac{R_m}{R_s} = \left(\frac{\rho_1^0 \alpha_1}{\rho_2^0} \gamma_g \left(\frac{u_0}{c_g} \right)^2 \right)^{1/n}. \tag{4.3}$$

Here ρ_1^0 and ρ_2^0 are the actual densities of the liquid and gas phase; and α_1 is the volumetric content of the liquid phase which corresponds to an inversion; and R_S is the radius of the liquid–air boundary at the moment the inversion front ceases. A more accurate estimate, which considers both the rate disequilibrium of the phases and the Rayleigh–Taylor instability at the external boundary of the liquid layer, leads to some correction factors in the last equation [29].

5. The solution to the problem of an explosion in a liquid layer surrounded by air breaks down spatially into three regions: the detonation products, the liquid, and the air. The total system of equations which describe the hydrodynamics and the cavitational disintegration (Secs. 1 and 2) is replaced by a system of finite-difference equations in a form analogous to [30], which has a second-order accuracy in the (spatial) coordinates and time. An artificial linear and quadratic viscous pressure is introduced to spread out the

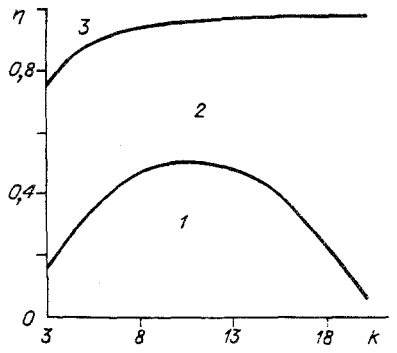

Fig. 3

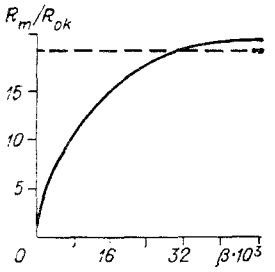

Fig. 4

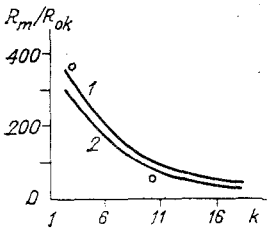

Fig. 5

hydrodynamic discontinuities in a single-pass calculation. The accuracy of the calculation is monitored via the conservation of the total energy of the system. In the calculation of the surface mass losses of liquid on the internal boundary of the liquid layer due to the development of the Rayleigh–Taylor instability, the initial conditions are taken to the values of the inner and outer radii of the liquid layer (R_c and R_k) and the pressure of the detonation products at the moment the inversion front ceases. Then, at each time step, calculations are done for the time to develop a perturbation (3.3), the velocity of the discontinuity in the perturbation layer of the liquid, and the removal of mass from the surface (3.4). Equation (3.4) is represented as a system of two first-order equations and is solved by a fourth-order Runge–Kutta method. The final dimension of the droplet–air cloud is determined from Eq. (4.3), in which we substitute the calculated dynamic parameters of the liquid layer at the moment the inversion front ceases. The calculations are done for $\rho_{ex} = 1.65 \cdot 10^3$ kg/m^3, D = 7600 m/sec, $p_{01} = p_{02} = 100$ kPa, $\rho_{02} = 1.18$ kg/m^2, $\gamma_g = 1.4$, $d_0 = 100$ μm, $\gamma_{liq} = 7$; for water: $\rho_{01} = 10^3$ kg/m^3; $c_{liq} = 1600$ m/sec; and $\sigma = 0.073$ N/m; for glycerin: $\rho_{01} = 1.26 \cdot 10^3$ kg/m^3; $c_{liq} = 1900$ m/sec; and $\sigma = 0.066$ N/m. The basic parameters of the liquid layer for spherical symmetry were taken as $R_{0c} = 5.1$ cm and $R_{0k} = 16.7$ cm; for cylindrical symmetry, $R_{0c} = 0.4$ cm and $R_{0k} = 2$ cm. By convention it is assumed that the moment of the inversion corresponds to an average volumetric bubble concentration of 50%. In this case, for an inversion which is understood as a structural transition from a bubble-filled liquid to a droplet–vapor mixture, no additional energy input is required, because at this moment the total surface of the interphase boundary of the liquid volume does not change (that is there is a phase conversion). Actually, the volumetric concentration of bubbles at the moment of inversion can exceed 50% [18], which, however, does not affect the calculated results, because long before the moment of inversion the liquid transforms to a state of free expansion. We note that the calculated results presented below are for water (unless mentioned otherwise).

The results of the numerical calculations show that after the shock wave reaches a free surface, a reflected rarefaction wave arises; behind its front a cavitation zone forms (p = 0). When the rarefaction wave reaches a gas bubble, as in [17], a jump in the acceleration of the adjacent liquid layer is observed, which leads to the agglomeration of the cavitational bubbles. Later, an inversion zone starts to propagate from the free surface of the liquid (more precisely from the rear boundary of the thin noncavitational "rind") into the gas bubble. On the inversion front, the bubble-filled liquid transforms into a droplet–vapor state. After the inversion front ceases in the Lagrangian coordinate system, the following picture is formed: the detonation products and the droplet mixture are separated by a noninverted layer of liquid which later disintegrates due to the Rayleigh–Taylor instability. Figure 1 shows the dimension d/d_0 of the droplet which forms during cavitational disintegration as a function of the Lagrangian coordinate r_0 during an explosion in a liquid spherical layer. Curves 1-3 correspond to values $k = R_{0k}/R_{0c} = 3.3$ and $\beta = 0.048$, $k = 3.3 \cdot \sqrt{2}$ and $\beta = 0.017$, and $k = 6.6$ and $\beta = 0.0066$. Here $\beta = m_{ex}/(m_{ex} + m_{liq})$ is the filling coefficient and m_{liq} is the mass of the liquid layer. From Fig. 1 it can be seen that the function d(r) has a monotonic character: the droplet dimension grows with the distance from the free surface of the liquid layer. Moreover, the increase in the filling coefficient β, which corresponds to a decrease in R_{0k} (m_{ex} = const), leads to a more detailed dispersion of the liquid layer: the average dimension of the droplets being formed decreases.

During an explosion in a cylindrical liquid layer, most attention is paid to the spatial dependence of the dimension of the "Rayleigh–Taylor" droplets which form during scattering

from the inner boundary of the liquid layer (Fig. 2), and also to the mass fraction η from the various disintegration mechanisms (Fig. 3, where 1 is the region of disintegration due to cavitation, 2 to the Rayleigh–Taylor instability, and 3 is the liquid layer). As can be seen from Fig. 3, the ratio k is determined by the disintegration process of the liquid layer. For k = 10, the mass fraction of the liquid layer disintegrated by cavitation reaches a maximum. As k increases further, disintegration due to the Rayleigh–Taylor instability becomes dominant. We emphasize again that the remaining liquid layer separates droplets formed by cavitation from the "Rayleigh–Taylor" droplets. This layer also decays into droplets when it reaches a critical thickness at $t = t_{**} > t_*$ (on the order of the average distance between the bubbles which have grown in the liquid).

Figure 4 shows the maximum expansion radius of the cloud R_m as a function of the filling coefficient β. The calculations were done for an explosion in a spherical volume of liquid. The horizontal dashed line shows the final dimension of the cloud of the detonation products which expand in an explosion in air [14]. A decrease in β corresponds to an increase in R_{0k}. As $\beta \to 0$, there is an initial liquid layer of infinite thickness, in which the rarefraction wave, which arises from reflection at the free surface, does not create a cavitation zone (and therefore also inversion regions) due to the negligibly small amplitude of the rarefaction wave.

Figure 5 shows the final dimension of the droplet–air cloud as a function of k in cylindrical geometry (curve 1 is for water and 2 is for glycerin). The choice of liquids and geometries in these calculations was determined from results of [15], in which an experimental function R(t) was obtained for these liquids for k = 3 and 10 (for each k the curves for water and glycerin were rather close, therefore a single average value can be used). By using the asymptotic values of R(t) (which determine the dimension of the droplet–air cloud at given experiment times [15]) as the final radii and by using the similarity principle with respect to $\sqrt{W}$, we select a value of the specific energy evolution e_{ex} [16], so that in Fig. 5 the points are shown relative to the calculated optimal curves. These points correspond to $e_{ex} = 25$ J/g. We note that the numerical calculations give a larger final dimension of the water cloud than does the experiment. The experiments determine the following time dependence: at the beginning of the observations ($t_1 \approx 5$ msec) the water cloud is larger than the glycerin cloud; then at $t_2 \approx 2$ msec these curves intersect; and at the end of the observation ($t_3 \approx 5$ msec) the dimension of the glycerin cloud is larger than the water cloud. The divergence can be explained as follows. The results of the numerical calculation show that at the moment the inversion front ceases, the dimension of the water cloud is larger than the glycerin cloud, which agrees with the experiment. Further on in the calculations Eq. (4.3) is used, that is, the stage of the expansion of the droplet cloud is not considered. Thus, a more correct comparison of the numerical results with experiments supposes more complete information on the experiments on one hand and on the other that the expansion of the droplet cloud should be calculated within the framework of two-phase hydrodynamics with two velocities (as in [21], for example).

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